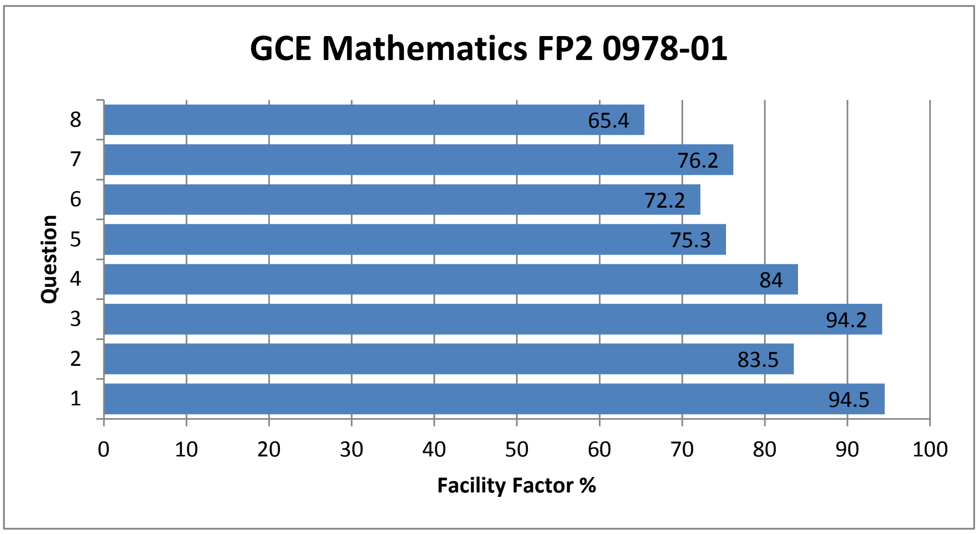
WJEC 2014 Online Exam Review

GCE Mathematics FP2 0978-01

All Candidates' performance across questions

Question Title	N	Mean	S D	Max Mark	F F	Attempt %	
1	412	6.6	1.1	7	94.5	100	
2	409	4.2	1.4	5	83.5	99.3	2
3	409	5.7	1	6	94.2	99.3	
4	410	6.7	2.2	8	84	99.5	4b
5	409	6	2.7	8	75.3	99.3	
6	393	5.8	2.9	8	72.2	95.4	
7	411	11.4	3.8	15	76.2	99.8	7b (iii)
8	408	11.8	5.2	18	65.4	99	8 a and b



2. Using the substitution $u = \sin^2 x$, evaluate the integral

$$\int_0^{\frac{\pi}{2}} \frac{\sin 2x}{\sqrt{4 - \sin^4 x}}\, dx.$$

Give your answer in the form $\frac{\pi}{k}$, where k is a positive integer. [5]

2. $$\int_0^{\pi/2} \frac{\sin 2x}{\sqrt{4-\sin^4 x}}\,dx$$

let $u = \sin^2 x$

$\frac{du}{dx} = \cos^2 x = (1-\sin^2 x)$

$= 1-u$

$dx = \frac{du}{1-u}$

When $x = \frac{\pi}{2}$, $u = \sin^2\left(\frac{\pi}{2}\right) = 1$

$\sin 2x = 2\sin x + 2\cos x$

When $x = 0$, $u = \sin^2(0) = 0$.

$\sin^4 x = u^2$

$$\int_0^1 \frac{\frac{du}{(1-u)}}{\sqrt{4-u^2}} = \int_0^1 \frac{\frac{du}{(1-u)}}{2-u}$$

2.

$$\int_0^{\pi/2} \frac{\sin 2x}{\sqrt{4-\sin^4 x}}\, dx$$

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M 1

$$\int_0^1 \frac{1}{\sqrt{4-u^2}} \cdot \frac{du}{(1-u)} = \int_0^1 \frac{1}{2-u} \cdot \frac{du}{(1-u)}$$

2

4. The complex number z is given by $1 + \mathrm{i}\sqrt{3}$.

(a) Find the modulus and the argument of z. [2]

(b) Find the three cube roots of z, giving your answers in the form $x + \mathrm{i}y$ with x and y correct to three decimal places. [6]

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4 b)	$z^3 = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$	
	$z_1 = 2^{1/3}\left(\cos\frac{\pi}{9} + i\sin\frac{\pi}{9}\right)$	
	$z_1 = 1.184 + 0.431i$	
	$z_2 = 2^{1/3}\left(\cos\frac{5\pi}{9} + i\sin\frac{5\pi}{9}\right)$	
	$z_2 = -0.219 + 1.241i$	
	$z_3 = 2^{1/3}\left(\cos\pi + i\sin\pi\right)$	
	$z_3 = -1.260$	

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$$z_1 = 1.184 + 0.431i$$ ✓

2

$$z_2 = 2^{1/3}\left(\cos\frac{5\pi}{9} + i\sin\frac{5\pi}{9}\right)$$ M0

$$z_2 = -0.219 + 1.241i$$

$$z_3 = 2^{1/3}\left(\cos\pi + i\sin\pi\right)$$ M0

4

$$z_3 = -1.260$$

7. The ellipse *E* has equation

$$4x^2 + 9y^2 = 36.$$

(a) Find

(i) the eccentricity,

(ii) the coordinates of the foci. [4]

(b) (i) Show that the point $P(3\cos\theta\ ,\ 2\sin\theta)$ lies on *E* for all values of θ.

(ii) Show that the equation of the tangent to *E* at *P* is

$$3y\ \sin\theta + 2x\ \cos\theta = 6.$$

(iii) This tangent meets the x-axis at *R* and the y-axis at *S*. The midpoint of *RS* is denoted by *M*. Determine the equation of the locus of *M* as θ varies. [11]

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7(b)(iii)

$$\frac{\frac{3}{\cos\theta}+0}{2} \quad \frac{\frac{2}{1\sin\theta}+0}{2}$$

$$=\left[\frac{3}{2\cos\theta} + \frac{1}{\sin\theta}\right]$$

$x = \frac{3}{2\cos\theta}$ $\quad u = 2\cos\theta,\ u' = -2\sin\theta$
$\quad v = 3 \quad v' = 0$

$$\frac{dx}{d\theta} = \frac{-6\sin\theta}{9}$$

$$\frac{d\theta}{dx} = \frac{-9}{6\sin\theta}$$

$$y = \frac{1}{\sin\theta} = (\sin\theta)^{-1}$$

$$\frac{dy}{d\theta} = -1(\cos\theta)^{-2}$$

$$\frac{dy}{dx} = \frac{9}{6\sin\theta\cos^2\theta}$$

$$y - \frac{1}{\sin\theta} = \frac{9}{6\sin\theta\cos^2\theta}\left(x - \frac{3}{2\cos\theta}\right)$$

~~$6y\sin\theta$~~

$\cos^2\theta = 1 - \sin^2\theta$

$6\sin\theta\cos^2\theta = (6\sin\theta - 6\sin^3\theta)$

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$\cos^2\theta = 1 - \sin^2\theta$

$6\sin\theta\cos^2\theta = (6\sin\theta - 6\sin^3\theta)$

3

8. The function f is defined by

$$f(x) = \frac{(x+4)(x-2)}{(x-4)}.$$

(a) Write down the coordinates of the points of intersection of the graph of f and the coordinate axes. [1]

(b) Determine the equation of

(i) the vertical asymptote on the graph of f,

(ii) the asymptote that is not parallel to a coordinate axis. [4]

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8, $f(x) = \dfrac{(x+4)(x-2)}{(x-4)}$

a) when intersecting the y axis $x = 0$

$f(0) = \dfrac{4 \times -2}{-4}$

$= 2$

intersects y axis at $(0,2)$

when intersecting the x axis $y = 0$

$0 = \dfrac{(x+4)(x-2)}{(x-4)}$

$x^2 - 2x + 4x - 8 = 0$

$x^2 + 2x - 8 = 0$

$(x-2)(x+4) = 0$

$x = 2 \qquad x = -4$

intersects x axis at $(2,0)$ and $(-4,0)$

b) vertical asymptote when $x - 4 = 0$

$x - 4 = 0$

$x = 4$

ii) $f(x) = \dfrac{x^2 + 2x - 8}{x - 4}$

$= \dfrac{x + 2 - 8/x}{1 - 4/x}$

$x \to \infty \qquad f(x) = x + 2$

asymptote $y = x + 2$.

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$x \to \infty \quad f(x) = x + 2$

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